

NAG Toolbox for MATLAB

f08fd

1 Purpose

f08fd computes selected eigenvalues and, optionally, eigenvectors of a real n by n symmetric matrix A . Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

2 Syntax

```
[a, m, w, z, isuppz, info] = f08fd(jobz, range, uplo, a, vl, vu, il, iu,
    abstol, 'n', n)
```

3 Description

The symmetric matrix is first reduced to a tridiagonal matrix T , using orthogonal similarity transformations. Then whenever possible, f08fd computes the eigenspectrum using Relatively Robust Representations. f08fd computes eigenvalues by the **dqds** algorithm, while orthogonal eigenvectors are computed from various ‘good’ LDL^T representations (also known as Relatively Robust Representations). Gram–Schmidt orthogonalisation is avoided as far as possible. More specifically, the various steps of the algorithm are as follows. For the i th unreduced block of T :

- compute $T - \sigma_i I = L_i D_i L_i^T$, such that $L_i D_i L_i^T$ is a relatively robust representation,
- compute the eigenvalues, λ_j , of $L_i D_i L_i^T$ to high relative accuracy by the dqds algorithm,
- if there is a cluster of close eigenvalues, ‘choose’ σ_i close to the cluster, and go to ,
- given the approximate eigenvalue λ_j of $L_i D_i L_i^T$, compute the corresponding eigenvector by forming a rank-revealing twisted factorization.

The desired accuracy of the output can be specified by the parameter **abstol**. For more details, see Dhillon 1997 and Parlett and Dhillon 2000.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

Barlow J and Demmel J W 1990 Computing accurate eigensystems of scaled diagonally dominant matrices *SIAM J. Numer. Anal.* **27** 762–791

Demmel J W and Kahan W 1990 Accurate singular values of bidiagonal matrices *SIAM J. Sci. Statist. Comput.* **11** 873–912

Dhillon I 1997 A new On^2 algorithm for the symmetric tridiagonal eigenvalue/eigenvector problem *Computer Science Division Technical Report No. UCB//CSD-97-971* UC Berkeley

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Parlett B N and Dhillon I S 2000 Relatively robust representations of symmetric tridiagonals *Linear Algebra Appl.* **309** 121–151

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz** – string

If **jobz** = 'N', compute eigenvalues only.

If **jobz** = 'V', compute eigenvalues and eigenvectors.

Constraint: **jobz** = 'N' or 'V'.

2: **range** – string

If **range** = 'A', all eigenvalues will be found.

If **range** = 'V', all eigenvalues in the half-open interval $(\mathbf{vl}, \mathbf{vu}]$ will be found.

If **range** = 'I', the **il**th to **iuth** eigenvalues will be found.

For **range** = 'V' or 'I' and $\mathbf{iu} - \mathbf{il} < \mathbf{n} - 1$, f08jj and f08jk are called.

Constraint: **range** = 'A', 'V' or 'I'.

3: **uplo** – string

If **uplo** = 'U', the upper triangular part of A is stored.

If **uplo** = 'L', the lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

4: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n symmetric matrix A .

If **uplo** = 'U', the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.

5: **vl** – double scalar

6: **vu** – double scalar

If **range** = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.

If **range** = 'A' or 'I', **vl** and **vu** are not referenced.

Constraint: if **range** = 'V', $\mathbf{vl} < \mathbf{vu}$.

7: **il** – int32 scalar

8: **iu** – int32 scalar

If **range** = 'I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned.

If **range** = 'A' or 'V', **il** and **iu** are not referenced.

Constraints:

if $\mathbf{n} = 0$, $\mathbf{il} = 1$ and $\mathbf{iu} = 0$;

if $\mathbf{n} > 0$, $1 \leq \mathbf{il} \leq \mathbf{iu} \leq \mathbf{n}$.

9: **abstol – double scalar**

The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a, b]$ of width less than or equal to

$$\mathbf{abstol} + \epsilon \max(|a|, |b|),$$

where ϵ is the *machine precision*. If **abstol** is less than or equal to zero, then $\epsilon \|T\|_1$ will be used in its place, where T is the tridiagonal matrix obtained by reducing A to tridiagonal form. See Demmel and Kahan 1990.

If high relative accuracy is important, set **abstol** to `x02am( )`, although doing so does not currently guarantee that eigenvalues are computed to high relative accuracy. See Barlow and Demmel 1990 for a discussion of which matrices can define their eigenvalues to high relative accuracy.

5.2 Optional Input Parameters1: **n – int32 scalar**

Default: The first dimension of the array **a** and the second dimension of the array **a**. (An error is raised if these dimensions are not equal.)

n , the order of the matrix A .

Constraint: $n \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

`lda, ldz, work, lwork, iwork, liwork`

5.4 Output Parameters1: **a(lda,*) – double array**

The first dimension of the array **a** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, n)$

The lower triangle (if **uplo** = 'L') or the upper triangle (if **uplo** = 'U') of **a**, including the diagonal, is destroyed.

2: **m – int32 scalar**

The total number of eigenvalues found.

If **range** = 'A', $m = n$.

If **range** = 'V', the exact value of **m** is not known in advance, but will satisfy $0 \leq m \leq n$.

If **range** = 'I', $m = iu - il + 1$.

3: **w(*) – double array**

Note: the dimension of the array **w** must be at least $\max(1, n)$.

The first **m** elements contain the selected eigenvalues in ascending order.

4: **z(ldz,*) – double array**

The first dimension, **ldz**, of the array **z** must satisfy

if **jobz** = 'V', $\mathbf{ldz} \geq \max(1, n)$;
 $\mathbf{ldz} \geq 1$ otherwise.

The second dimension of the array must be at least $\max(1, m)$

If **jobz** = 'V', then if **info** = 0, the first m columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i th column of Z holding the eigenvector associated with $\mathbf{w}(i)$.

If **jobz** = 'N', $\mathbf{z}$ is not referenced.

Note: you must ensure that at least $\max(1, \mathbf{m})$ columns are supplied in the array $\mathbf{z}$; if **range** = 'V', the exact value of M is not known in advance and an upper bound must be used.

5: **isuppz**(*) – int32 array

Note: the dimension of the array **isuppz** must be at least $\max(1, 2 \times \mathbf{m})$.

The support of the eigenvectors in $\mathbf{z}$, i.e., the indices indicating the nonzero elements in $\mathbf{z}$. The i th eigenvector is nonzero only in elements **isuppz**($2 \times i - 1$) through **isuppz**($2 \times i$). Implemented only for **range** = 'A' or 'I' and **iu** – **il** = **n** – 1.

6: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **range**, 3: **uplo**, 4: **n**, 5: **a**, 6: **lda**, 7: **vl**, 8: **vu**, 9: **il**, 10: **iu**, 11: **abstol**, 12: **m**, 13: **w**, 14: **z**, 15: **ldz**, 16: **isuppz**, 17: **work**, 18: **lwork**, 19: **iwork**, 20: **liwork**, 21: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

f08fd failed to converge.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the *machine precision*. See Section 4.7 of Anderson *et al.* 1999 for further details.

8 Further Comments

The total number of floating-point operations is proportional to n^3 .

The complex analogue of this function is f08fr.

9 Example

```
jobz = 'Vectors';
range = 'I';
uplo = 'Upper';
a = [1, 2, 3, 4;
     0, 2, 3, 4;
     0, 0, 3, 4;
```

```

    0, 0, 0, 4];
vl = 0;
vu = 0;
il = int32(2);
iu = int32(3);
abstol = 0;
[aOut, m, w, z, isuppz, info] = f08fd(jobz, range, uplo, a, vl, vu, il,
iu, abstol)

aOut =
   -0.3571    0.1237    0.6262    0.3660
         0   -0.9762   -1.2472    0.3660
         0         0    7.3333   -6.9282
         0         0         0    4.0000

m =
      2

w =
   -0.5146
   -0.2943
         0
         0

z =
   -0.5144    0.2767
    0.4851   -0.6634
    0.5420    0.6504
   -0.4543   -0.2457

isuppz =
         0
         0
         0
         0
         0
         0
         0
         0
         0

info =
         0

```